ERRATUM TO LIFTABLE MAPPING CLASS GROUP OF BALANCED SUPERELLIPTIC COVERS

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There is an error in the statement (but not the proof) of Lemma 6.3. It should be as follows.

Lemma 6.3. *In the abelianization of* G_n*,* $B^{n^2} = A^{-n^2-n}$.

This typo has a follow-on effect for the rest of the paper. The statement of Lemma 6.5 should be as follows, although the proof still holds as written.

Lemma 6.5. *The abelianization* $G_n/[G_n, G_n]$ *admits the presentation*

$$\left\langle a, d, A, B \mid B^{n^2-n} = A^{1-n^2}, B^{n^2} = A^{-n^2-n}, a^2 = B, d^2 = A^{n+1}, \mathcal{T} \right\rangle$$

where $a = \phi(a_1)d = \phi(c), A = \phi(A_{12}), B = \phi(A_{13})$*, and* $\mathcal{T}$ *is the set of all commutators.*

Theorem 1.1 now has a different statement and proof, which we present now.

Theorem 1.1. *Let* $k \geqslant 3$*. Then*

$$H_1(\mathrm{LMod}_{g,k}(\Sigma_0, \mathcal{B})) \cong \begin{cases} \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z} & \text{if } n \text{ is odd,} \\ \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z} & \text{if } n \text{ is even.} \end{cases}$$

Proof. Replacing the generator B with a^2 gives the presentation

$$\left\langle a, d, A \mid a^{2n^2-2n} = A^{1-n^2}, a^{2n^2} = A^{-n^2-n}, d^2 = A^{n+1}, \mathcal{T} \right\rangle$$

for $G_n/[G_n, G_n]$. This presentation has presentation matrix

$$\begin{bmatrix} 2 & -n-1 & 0 \\ 0 & n^2-1 & 2n^2-2n \\ 0 & n^2+n & 2n^2 \end{bmatrix}.$$

If n is odd we can perform row and column operations below to obtain the Smith normal form:

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

. Therefore $G_n/[G_n, G_n] \cong \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}$.

If n is even, the Smith normal form will be:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Therefore $G_n/[G_n, G_n] \cong \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}$. $\square$

Of course, the correct statement of Theorem 1.1 implies the following correct statement of Theorem 1.2.

Theorem 1.2. *The abelianization of the balanced superelliptic mapping class group* $H_1(\mathrm{SMod}_{g,k}(\Sigma_g); \mathbb{Z})$ *is an infinite non-cyclic abelian group. Furthermore, the first Betti number of* $\mathrm{SMod}_{g,k}(\Sigma_g)$ *is 1.*

We would like to thank Michael Lönne for pointing out the error in Lemma 6.3 and the subsequent incongruence in results.

References

[1] Tyrone Ghaswala and Rebecca R. Winarski. The liftable mapping class group of superelliptic covers. *New York Journal of mathematics*, 23:133–164, 2017.